MA 425 - Quiz #3 NAME Key

① State (don't prove) the Archimedean Property.

If $x \in \mathbb{R}$ $\exists$ $n_x \in \mathbb{N}$ $\ni$ $x \le n_x$

② Show that if $\varepsilon \in \mathbb{R}$, $\varepsilon > 0$, then $\exists$ $n \in \mathbb{N}$ $\ni$ $\frac{1}{n} < \varepsilon$.

$\frac{1}{\varepsilon} \in \mathbb{R}$ so $\exists$ $n \in \mathbb{N}$ $\ni$ $\frac{1}{\varepsilon} < n$ by Arch. Prop.

$\frac{1}{\varepsilon} < n \iff \frac{1}{n} < \varepsilon$.

③ Suppose that $S \subset \mathbb{R}$ is nonempty and bounded above, that $a > 0$, and $T = \{as : s \in S)$ (T is often denoted aS). Show

$$\sup(T) = a\sup(S)$$

Let $u = \sup(S)$. If $t \in T$ then $\exists$ $s \in S$ $\ni$

$t = as \Rightarrow t \le au$ since $a > 0$ and $s \le u = \sup(S)$.

$\therefore$ au is an ub for T.

Suppose v is an ub for T. ~~Then~~ If $t = as \in T$, then

$as \le v \Rightarrow s \le \frac{v}{a}$.

$\therefore$ $s \le \frac{v}{a}$ $\forall$ $s \in S$ so $u \le \frac{v}{a} \iff v \ge au$.

$\therefore$ au is the lub ~~of~~ for T so $\sup(T) = a\sup(S)$.