MA 425 - 1st Quiz - NAME Key

① Suppose $f: A \to B$, $g: B \to C$ and $g \circ f: A \to C$ is onto. Show g is onto.

Let $z \in C$. Need to find $y \in B$ s.t. $g(y) = z$ (i.e. Show g is onto)

Given: $g \circ f$ is onto so $\exists\ x \in A$ s.t. $g \circ f(x) = z \Rightarrow$

$g(f(x)) = z$.

$\therefore$ If $y = f(x)$ then $g(y) = z$ so g is onto.

② Suppose $f(x) = \frac{2x}{x-4}$ $\forall\ x \in \mathbb{R}, x \neq 4$. Show f is 1-1 on $(-\infty, 4) \cup (4, \infty)$.

Suppose $x_1, x_2 \in (-\infty, 4) \cup (4, \infty)$ and $f(x_1) = f(x_2)$.

need to show $x_1 = x_2$ (i.e f is 1-1).

$f(x_1) = f(x_2) \Rightarrow \frac{2x_1}{x_1 - 4} = \frac{2x_2}{x_2 - 4}$ ● ($x_1 \neq 4$ and $x_2 \neq 4$)

$\Rightarrow 2x_1(x_2 - 4) = 2x_2(x_1 - 4)$

$\Rightarrow 2x_1x_2 - 8x_1 = 2x_2x_1 - 8x_2$

$\Rightarrow -8x_1 = -8x_2$

$\Rightarrow x_1 = x_2$ ✓

$\therefore$ f is 1-1.